Supplementary Material

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In our numerical simulations, we opt to initialize the system using the exact (numerically identified) standing wave solution, in each example, perturbed by adding the internal mode profile (multiplied by a prefactor smaller than unity). Although, "in principle", projection of truncation errors on the internal mode may be eventually amplified and manifest the instability, we have not followed this avenue here, chiefly because this may take very long, especially for such a nonlinear instability. Instead, what we do here is that we actively perturb along the direction of the internal mode, using a nontrivial such perturbation. Our numerical experiments have shown that a sizable such perturbation (e.g. of the order of 10% of the mode amplitude in Fig. 1 –or slightly less, e.g., in Figs. 2-3–) enables the more rapid (and within our numerically accessible time frame) observation of the pertinent nonlinear instability.

The system is then marched forward with a Runge-Kutta integrator of high order (typically 4th order), with a sufficiently small time step (typically 10^{-3} or less), while the energy, as well as norm (i.e., power) conservation are monitored as diagnostics of the accuracy of the numerical computation.

For completeness, we also show here a prototypical example of the spectral picture associated with the scenario of Fig. 1 of the main text. In particular, the linearization spectrum around the anti-symmetric (first excited) state of the double well potential is shown in Fig. 1. There, it is corroborated that while the spectrum of linear (extended) modes starts at $\lambda = \pm 0.4i$, the internal mode illustrated at $0.203i$ has a second harmonic (shown by a red square) which is inside the continuous spectrum. Thus, the resonance is active and results in the nonlinear instability observed in the text.

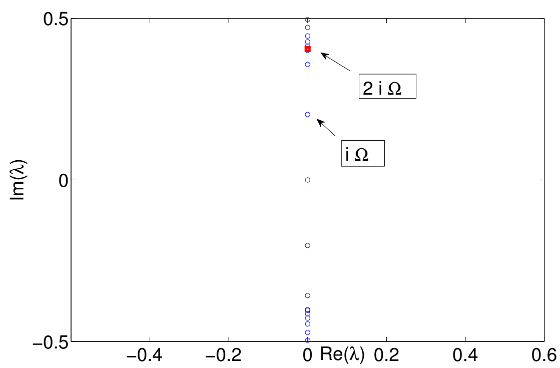

FIG. 1: For the case of $\omega = -0.4$ in the double well potential of $x_0 = 2$ and $V_0 = -1$, the linearization spectrum around the anti-symmetric solution is shown. Clearly depicted is the relevant internal mode with $\lambda = 0.203i$ and its second harmonic (shown by a red square) which is resonant with the linear mode spectrum.