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pp. **i–iii**

PREFACE: RECENT PROGRESS ON THE LONG TIME BEHAVIOR OF COHERENT STRUCTURES IN DISCRETE AND CONTINUOUS MODELS

Partial differential equations viewed as dynamical systems on an infinite-dimensional space describe many important physical phenomena. Lately, an unprecedented expansion of this field of mathematics has found applications in areas as diverse as fluid dynamics, nonlinear optics and network communications, combustion and flame propagation, to mention just a few. In addition, there have been many recent advances in the mathematical analysis of differential difference equations with applications to the physics of Bose-Einstein condensates, DNA modeling, and other physical contexts. Many of these models support coherent structures such as solitary waves (traveling or standing), as well as periodic wave solutions. These coherent structures are very important objects when modeling physical processes and their stability is essential in practical applications. Stable states of the system attract dynamics from all nearby configurations, while the ability to control coherent structures is of practical importance as well. This special issue of Discrete and Continuous Dynamical Systems is devoted to the analysis of nonlinear equations of mathematical physics with a particular emphasis on existence and dynamics of localized modes. The unifying idea is to predict the long time behavior of these solutions. Three of the papers deal with continuous models, while the other three describe discrete lattice equations.

Genev and Venkov study the Cauchy problem for the focusing time-dependent Schrödinger-Hartree equation $i\partial_t\psi + \triangle\psi = -(|x|^{-(n-2)} * |\psi|^{\alpha})|\psi|^{\alpha-2}\psi, \quad \alpha \geq 2$, for space dimension $n \geq 3$. They prove the existence of solitary wave solutions and give conditions for formation of singularities in dependence of the values of $\alpha \geq 2$ and the initial data $\psi(0,x) = \psi_0(x)$. The dispersive properties of the Schrödinger operator and Strichartz estimates, applied to the Duhamel's integral representation, are the basic tools in establishing the local and global existence results, while variance identities are used to construct blow-up solutions in the supercritical case. The authors present a deeper analysis of the blow-up phenomenon, based on the compactness theorem and the mass concentration phenomenon, observed at the mass-critical level.

The sine-Gordon equation $u_{tt} = u_{xx} + \sin u$ and the spectral stability of its kink waves is the subject of the Jones and Marangell paper. The authors take a more geometric, dynamical systems approach. Kink-wave solutions correspond to critical points of a nonlinear dynamical system on an appropriate Hilbert space. They consider the linearized operator about such a critical point corresponding to a traveling wave solution to equation and investigate the spectrum of this linear operator. Using various geometric techniques as well as some elementary methods from ODE theory, they find that the point spectrum of this operator is purely imaginary provided the wave speed c of the traveling wave is not ± 1. The authors use a crossing form calculation in order to show the absence of real point spectrum of the linearized operator. The crossing form is inspired from the Maslov index techniques. The essential spectrum of the operator in the subluminal case lies entirely on the imaginary axis, while in the superluminal case it consists of an ellipse centered at the origin, intersecting the real axis at ± 1, and intersecting the imaginary axis at $\pm c$, together with a pair of lines which are contained in the imaginary axis, but extend past the vertices of the ellipse on the imaginary axis to the points $\pm i\sqrt{c^2-1}$.

The paper by Latushkin and Sukhtayev establishes connections between the classical Weyl-Titchmarsh function for Hamiltonian ordinary differential equations, and the Evans function, a Wronskian-type determinant designed to detect point spectrum of ordinary differential operators arising in the study of stability of traveling waves and other patterns for partial differential equations. The authors give a number of examples for which their general results are applicable. As a byproduct, for the scalar Schrödinger equation, the authors give a formula relating the Evans function for problems on the full line to that on finite segments. An interesting and still open question is if this "product formula" holds for more general classes of equations.

Mizumachi and Pelinovsky consider the discrete nonlinear Schrödinger (DNLS) equation with a power nonlinearity and a bounded potential, $i\dot{u}_n = (-\Delta + V_n + |u_n|^{2p})u_n, \quad n \in \mathbf{Z}$, where $p > 0$, $\{V_n\}_{n\in\mathbf{Z}} \in l^\infty(\mathbf{Z})$, and $\Delta u_n := u_{n+1} - 2u_n + u_{n-1}$. A localized mode is a real-valued solution of the stationary DNLS equation, $(-\Delta + V_n + \phi_n^{2p})\phi_n = \omega\phi_n$. Asymptotic stability of localized modes in the DNLS equation was established earlier for septic and higher-order nonlinear terms by using Strichartz estimates. The authors use here pointwise dispersive decay estimates to push down the lower bound for the exponent of the nonlinear terms. Because the spectral assumption for the standard asymptotic stability theory is satisfied, it is natural to expect that the details of this work can be extended to the DNLS equation with $V \equiv 0$ near the anti-continuum limit, for the price of working with dispersive estimates for non-self-adjoint linearized operators.

Macroscopic wave packets in infinite chains of coupled oscillators, like the famous Fermi-Pasta-Ulam (FPU) system $\partial_t^2 q_n(t) = W'(q_{n+1}(t) - q_n(t)) - W'(q_n(t) - q_{n-1}(t)), \quad n \in \mathbf{Z}$, with $q_n(t) \in \mathbf{R}$ can be described by simple partial differential equations, like the Korteweg-de Vries (KdV) equation or the Nonlinear Schrödinger (NLS) equation. A key observation here is that by considering the discrete Fourier transform, the existing proofs for the justification of the NLS approximation of wave packets in spatially homogeneous dispersive PDE systems can be transferred almost line for line. The purpose of the paper by Martina Chirilus-Bruckner, Christopher Chong, Oskar Prill and Guido Schneider is to prove error estimates for the approximate description of macroscopic wave packets in infinite periodic chains of coupled oscillators by modulation equations, like the Korteweg–de Vries (KdV) or the Nonlinear Schrödinger (NLS) equation. The proofs are based on a discrete Bloch wave transform of the underlying infinite-dimensional system of coupled ODEs. After this transform the existing proof for the associated approximation theorem for the NLS approximation used for the approximate description of oscillating wave packets in dispersive PDE systems transfers. The proof of the approximation theorem for the KdV approximation of long waves is less obvious, but in a special situation a first approximation result is proved.

Ptitsyna and Shipman present a discrete model of resonant scattering of waves by an open periodic waveguide. The model elucidates a phenomenon common in electromagnetics, in which the interaction of plane waves with embedded guided modes of the waveguide causes sharp transmission anomalies and field amplification. The ambient space is modeled by a planar lattice and the waveguide by a linear periodic lattice coupled to the planar one along a line. The authors show the existence of standing and traveling guided modes and analyze a tangent bifurcation, in which resonance is initiated at a critical coupling strength where a guided mode appears, beginning with a single standing wave and splitting into a pair of waves traveling in opposing directions. Complex perturbation analysis of the scattering problem in the complex frequency and wavenumber domain reveals the complex structure of the transmission coefficient at resonance.

The normal referee procedure and standard practice of AIMS journals were followed for final acceptance of these papers. We wish to thank all the referees who generously devoted their time and effort reading and carefully checking all the papers and providing comments and recommendations. We wish to also thank AIMS for providing a platform to publish this special issue and for the technical help.

Guest Editors:
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