Kadomtsev–Petviashvili equation

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The *Kadomtsev-Petviashvili equation* (or simply the KP equation) is a nonlinear partial differential equation in two spatial and one temporal coordinate which describes the evolution of nonlinear, long waves of small amplitude with slow dependence on the transverse coordinate. There are two distinct versions of the KP equation, which can be written in normalized form as follows:

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma^2 u_{yy} = 0. \tag{1}$$

Here $u = u(x,y,t)$ is a scalar function, x and y are respectively the longitudinal and transverse spatial coordinates, subscripts x, y, t denote partial derivatives, and $\sigma^2 = \pm 1$. The case $\sigma = 1$ is known as the KPII equation, and models, for instance, water waves with small surface tension. The case $\sigma = i$ is known as the KPI equation, and may be used to model waves in thin films with high surface tension. The equation is often written with different coefficients in front of the various terms, but the particular values are inessential, since they can be modified by appropriately rescaling the dependent and independent variables.

The KP equation is a universal integrable system in two spatial dimensions in the same way that the KdV equation can be regarded as a universal integrable system in one spatial dimension, since many other integrable systems can be obtained as reductions. As such, the KP equation has been extensively studied in the mathematical community in the last forty years. The KP equation is also one of the most universal models in nonlinear wave theory, which arises as a reduction of system with quadratic nonlinearity which admit weakly dispersive waves, in a paraxial wave approximation. The equation naturally emerges as a distinguished limit in the asymptotic description of such systems in which only the leading order terms are retained and an asymptotic balance between weak dispersion, quadratic nonlinearity and diffraction is assumed. The different role played by the two spatial variables accounts for the asymmetric way in which they appear in the equation.

Despite their apparent similarity, the two versions of the KP equation differ significantly with respect to their underlying mathematical structure and the behavior of their solutions. Figure 1 shows a two-dimensional localized solution of the KPI equation, known as the lump solution, while Figure 2 shows a contour plot of a resonant two-soliton solution of the KPII equation. Solutions of these KP equations and their properties are discussed in more detail in the following sections.

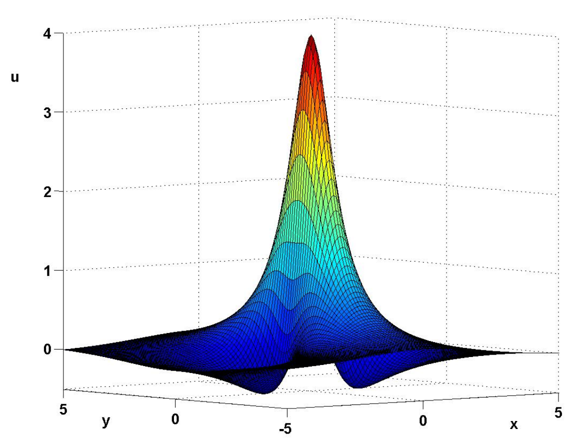

Figure 1: A lump solution of KPI.

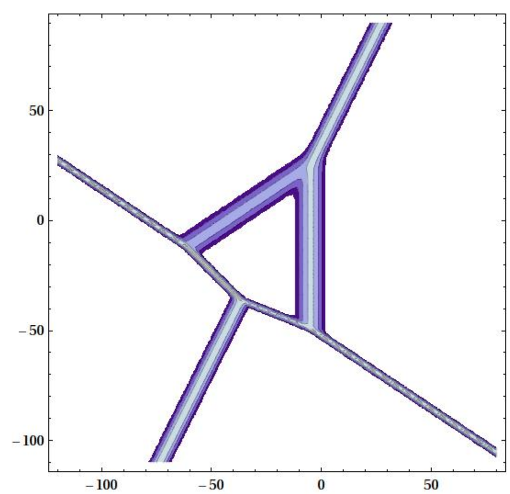

Figure 2: A resonant 2-soliton solution of KPII.

Contents

1. History

2. Mathematical structure

2.1 Lax pair and equivalent formulations

2.2 Bilinear form and Wronskian representations

2.3 Connection with Sato theory

3. Exact solutions and two-dimensional wave phenomena

3.1 Line solitons

3.2 Existence and stability of two-dimensional solitary waves in the KPI equation

3.3 Transverse stability of one-dimensional solitary waves

3.4 Resonant interactions of line solitons in the KPII equation

3.5 Finite-genus and quasi-periodic solutions

4 References

5 Recommended reading

History

The KP equation originates from a 1970 paper by two Soviet physicists, Boris Kadomtsev (1928-1998) and Vladimir Petviashvili (1936-1993). The two researchers derived the equation that now bears their name as a model to study the evolution of long ion-acoustic waves of small amplitude propagating in plasmas under the effect of long transverse perturbations. In the absence of transverse dynamics, this problem is described by the Korteweg-de Vries (KdV) equation. The KP equation was soon widely accepted as a natural extension of the classical KdV equation to two spatial dimensions, and was later derived as a model for surface and internal water waves by Ablowitz and Segur (1979), and in nonlinear optics by Pelinovsky, Stepanyants and Kivshar (1995), as well as in other physical settings.

The focus of the 1970 paper was on a particular problem, the stability of solitons of the Korteweg-de Vries equation with respect to transverse perturbations. The authors showed that KdV solitons are stable to such perturbations in the case of media characterized by negative dispersion (that is, when the phase speed of infinitesimal perturbations decreases with the wavenumber). This is the case of the KPII equation. In the opposite case of a positive dispersion media (where the phase speed increases with the wavenumber), however, KdV solitons are unstable. This is the case of the KPI equation.

The discovery of the KP equation happened almost simultaneously with the development of the inverse scattering transform (IST) [e.g., see Ablowitz and Segur (1981) and Novikov, Manakov, Pitaevski and Zakharov (1984)]. This method for the solution of the initial-value problem for nonlinear partial differential equations was originally developed for equations in one spatial dimension, such as the KdV equation. In 1974, however, Valery Dryuma showed how the KP equation could be written in Lax form, providing a strong hint that the equation was integrable. Then, in the same year, Vladimir Zakharov and Alexey Shabat extended the IST to equations in two spatial dimensions, including the KP equation, and obtained several exact solutions to the KP equation, including line-soliton solutions. A few years later, various researchers also obtained two-dimensional algebraically decaying localized solutions of the KPI equation, which are called lumps. These solutions allowed physicists to solve a number of theoretical problems involving the KP equation in the following twenty years, as discussed below.

Throughout the 1980s and 1990s, both versions of the KP equations were used as prototypical examples for further advances in the IST involving problems of complex analysis. In particular, the nonlocal Riemann-Hilbert problem and the D-Bar problem were applied to the KPI and KPII equations, respectively, as methods of solutions of the inverse scattering problem using integration in the complex plane. The relevant works of Mark Ablowitz, Athanassios Fokas and others are reviewed in the book by Ablowitz and Clarkson (1991), while an alternative point of view on the dbar-dressing method is presented in the book by Boris Konopelchenko (1993). The inverse scattering method relates solutions of the KPI and KPII equations to solutions of the time-dependent Schroedinger equation and the heat equation with external potential, respectively. Fundamental properties of Darboux-Backlund transformations for solutions of these equations are described in the book by Vladimir Matveev and Michael Salle (1991). It should be noted however that the implementation of IST in the case of non-vanishing boundary conditions at infinity has proved to be significantly more difficult, and it is still the subject of current research [e.g., see Boiti *et al.* (2002,2006) and Villarroel and Ablowitz (2002)].

The KP equation has been used extensively as a model for two-dimensional shallow water waves [Segur and Finkel (1985), Hammack *et al.* (1989,1995)] and ion-acoustic waves in plasmas [e.g., see Infeld and Rowlands (2001)]. More recently, it has been obtained as a reduced model in ferromagnetics, Bose-Einstein condensation and string theory. The KP equation is still used as a classical model for developing and testing of new mathematical techniques, e.g. in problems of well-posedness in non-classical function spaces [Tzvetkov (2000)], in applications of the dynamical system methods for water waves [Groves and Sun (2008)], and in the variational theory of existence and stability of energy minimizers [De Bouard and Saut (1997)].

Mathematical structure

Lax pair and equivalent formulations

A Lax pair for the KP equation is given by the overdetermined linear system

$$\sigma\psi_y + \psi_{xx} + (u+\lambda)\psi = 0\,, \tag{2}$$

$$\psi_t + 4\psi_{xxx} + 6u\psi_x + 3u_x\psi - 3\sigma\big(\partial^{-1}u_y\big)\,\psi + \alpha\psi = 0\,, \tag{3}$$

where the solution $u(x,y,t)$ of the KP equation plays the role of a scattering potential, $\psi(x,y,t,\lambda)$ is the corresponding eigenfunction, λ is the spectral parameter, α is an arbitrary constant, and

$$(\partial^{-1}f)(x) = \frac{1}{2}\Big(\int_{-\infty}^{x} f(x')dx' - \int_{x}^{\infty} f(x')dx'\Big). \tag{4}$$

The particular definition of the operator ∂^{-1} used here is convenient for IST, and allows one to write the KP equation in evolution form. That is, the compatibility of (2) and (3) requires that u satisfies

$$u_t + 6uu_x + u_{xxx} + 3\sigma^2\partial^{-1}u_{yy} = 0, \tag{5}$$

which is the KP equation (1) after integration with the operator ∂^{-1}. The same equation (5) is also often written in compatibility form as the system

$$u_t + 6uu_x + u_{xxx} = -3\sigma^2 v_x\,, \qquad v_x = u_y\,. \tag{6}$$

These two formulations are equivalent under suitable conditions of convergence and regularity. Note also that the validity of equation (5) requires the invariant constraint

$$\int_{-\infty}^{\infty} u_{yy}(x,y,t)\,dx = 0. \tag{7}$$

This condition imposes an infinite number of constraints on the initial datum. In fact, even if this constraint is not satisfied at $t=0$, the time evolution is such that a discontinuity develops at $t=0$, and the constraint is satisfied at all $t>0$ [see Ablowitz and Villarroel (1991)].

The IST for the KP equation is based on the spectral analysis of Eq. (2). Even though the Lax pairs of KPI and KPII are almost identical, however, the the IST is profoundly different. In particular, the IST for KPI employs a nonlocal Riemann-Hilbert problem, while that for KPII requires the use of the so-called dbar method [Ablowitz and Clarkson (1991), Konopelchenko (1993), Novikov ”et al” (1984)].

Bilinear form and Wronskian representations

The KP equation can be written in bilinear form by expressing solutions in terms of a tau-function. If

$$u(x,y,t) = 2\partial_x^2 \log \tau(x,y,t)\,, \tag{8}$$

then $\tau(x,y,t)$ satisfies the Hirota bilinear equation:

$$\left[D_x D_t + D_x^4 + 3D_y^2\right]\tau\cdot\tau = 0\,, \tag{9}$$

where D_x, D_y, D_t are Hirota derivatives:

$$D_x^m f\cdot g = (\partial_x - \partial_{x'})^m f(x,y,t)g(x',y,t)|_{x=x'}\,, \tag{10}$$

etc. This formulation provides the basis for using Hirota's method to obtain solutions of the KP equation [see Hirota (2004)].

Solutions to Hirota's bilinear equation can also be obtained by expressing the tau-function in Wronskian form [see Freeman and Nimmo (1983)]:

$$\tau(x,y,t) = \mathrm{Wr}(f_1,\ldots,f_N) = \det\begin{pmatrix} f_1^{(0)} & \cdots & f_N^{(0)} \\ \vdots & \ddots & \vdots \\ f_1^{(N-1)} & \cdots & f_N^{(N-1)} \end{pmatrix}, \tag{11}$$

where $f^{(n)}$ denotes the n¡sup¿th¡/sup¿ partial derivative of f with respect to x, and where the functions $f_1,\ldots,f_N$ solve the associated linear problem

$$\sigma f_y + f_{xx} = 0\,, \qquad f_t + 4f_{xxx} = 0\,. \tag{12}$$

This linear system represents the Lax pair (2)-(3) for $\lambda = \alpha = 0$ and $u \equiv 0$. A large variety of exact solutions of the KP equation can be obtained in this way. Some of them are briefly discussed below.

Connection with Sato theory

Equations solvable by the inverse scattering transform technique are members of an infinite hierarchy of commuting time flows. Originating from the works of the Japanese mathematician Mikio Sato in the 1980s, this point of view considers the KP equation as the first non-trivial example of the KP hierarchy, which is a hierarchy of nonlinear partial differential equations in an infinite number of independent and dependent variables [see Miwa "et al" (2000)]. If L is the pseudodifferential operator

$$L = \partial_x + \sum_{m=1}^{\infty} u_m \partial_x^{-m}, \tag{13}$$

where u_m, $m \geq 1$ are functions of $(t_1, t_2, t_3, ...)$, the equations of the KP hierarchy are equivalent to the generalized Lax equation

$$\frac{\partial L}{\partial t_n} = [B_n, L], \quad n \geq 1, \tag{14}$$

where B_n is the differential part (including any purely multiplicative terms) of L^n, so that $t_1 = x$ follows from the Lax equation above for $n = 1$. The KP equation is found from the generalized Lax equation for $n = 2$ and $n = 3$ with the correspondence $u = u_1$, $t_2 = \sigma y$ and $t_3 = -4t$. The pseudodifferential operator L was first used in the work of Gelfand and Dikii (1976).

The KP hierarchy supplemented with a number of classical and non-classical reductions contains many integrable equations, such as the Boussinesq and nonlinear Schroedinger equations. Moreover, the hierarchy can be extended even further to a two-component hierarchy of Davey-Stewartson equations, which themselves contain other integrable equations. This development has deviated far from the original scope of the Kadomtsev-Petviashvili equation. As a result, many researchers nowadays use the KP equation without knowing where the equation originated and what "K" and "P" stand for.

Line solitons

The one-soliton solution of KP is

$$u(x,y,t) = \tfrac{1}{2}a^2 \operatorname{sech}^2\left[\tfrac{1}{2}a\,(\,x - by - \omega t/a - x_0\,)\right], \tag{15}$$

where (a,b,x_0) are arbitrary parameters, whereas ω depends on (a,b). Equation (15) is a traveling wave solution: $u(x,y,t) = U(\mathbf{k}\cdot\mathbf{x} - \omega t)$, where $\mathbf{x} = (x,y)$ and $\mathbf{k} = (k_x,k_y) = (a,-ab)$, with its peak localized along the moving line $\mathbf{k}\cdot\mathbf{x} = \omega t$. Thus, the solution is referred to as a ”line soliton” (or plane soliton), which is tantamount to a tilted version of the KdV soliton. Apart from the arbitrary translation constant x_0, the solution (15) depends on two parameters: the ”soliton amplitude” a and the ”soliton direction” b (that is, the soliton inclination in the xy-plane: $b = \tan\alpha$, where α is the angle from the positive y-axis, measured counterclockwise). The ”soliton frequency” ω is given by the soliton dispersion relation $D(\mathbf{k},\omega) = 0$, where

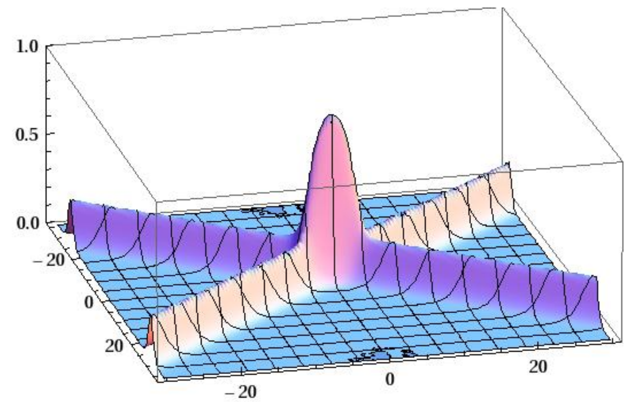

Figure 3: An ”ordinary” 2-soliton solution of KPII.

$$D(\mathbf{k},\omega) = 4\omega\, k_x + k_x^4 - 3\sigma^2 k_y^2\,. \tag{16}$$

Generalizations of the above to N-soliton solutions exist, and they describe the interactions of N line solitons. In the simplest cases, these solutions produce a pattern of N intersecting lines in the xy-plane, together with small interaction phase shifts. For example, Figure 3 shows such a 2-soliton solution. [See also the famous photo of interacting water waves off the Oregon coast by Terry Toedtemeier.] For the KPI equations, however these solutions are all unstable. For the KPII equation they are believed to be stable, although no formal proof exists. Moreover, more general multi-soliton solutions also exist for the KPII equation and describe phenomena of soliton resonance and web structure, as discussed below.

Existence and stability of two-dimensional solitary waves in the KPI equation

The KPI equation has a two-dimensional solitary wave called a lump. It is given by a simple rational expression in the independent variables:

$$u(x,y,t) = 4\frac{-(x + ay + 3(a^2 - b^2)t)^2 + b^2(y + 6at)^2 + 1/b^2}{\left[(x + ay + 3(a^2 - b^2)t)^2 + b^2(y + 6at)^2 + 1/b^2\right]^2}, \tag{17}$$

where (a,b) are real parameters. Such a solution is shown in Figure 1 above.

The exact analytical form of the lump solution given above was found in 1977 by Manakov *et al.*, who also studied the interaction of lumps and found that these interactions do not result in a phase shift as in the case of line solitons. Subsequently, various researchers obtained more general rational solutions of the KPI equation [e.g. see Krichever (1978), Satsuma and Ablowitz (1979), Pelinovsky and Stepanyants (1993) and Pelinovsky (1994)]. These solutions were then reconciled with the framework of the IST by Villarroel *et al.* (1999). They found that, in general, the spectral characterization of the potential must include, in addition to the usual information about discrete and continuous spectrum, an integer-valued topological quantity that they called the ”index” or winding number, defined by an appropriate two-dimensional integral involving both the solution of the KP equation and the corresponding scattering eigenfunction.

No real non-singular rational solutions are known for KPII.

Transverse stability of one-dimensional solitary waves

It was known since the original work by Kadomtsev and Petviashvili that, with respect to long transverse perturbations one-dimensional solitons are stable in the KPII equation and unstable in the KPI. Evolution of transverse perturbations of arbitrary scale for KP1 was first investigated by [Zakharov (1975)], where stabilization for large wave numbers was found. This finding helped to demonstrate that the instability results in the break-up of a one-dimensional soliton into a periodic chain of two-dimensional solitons [see Pelinovsky and Stepanyants (1993)]. The corresponding solution is given by the tau-function (8) with

$$\tau = 1 + e^{2p\eta} + c^2 e^{2q\zeta} + \frac{4\sqrt{pq}}{p+q} c \cos((p^2 - q^2)y) e^{(p+q)\xi}, \tag{18}$$

where

$$\eta = x - p^2 t, \quad \zeta = x - q^2 t, \quad \xi = x - (p^2 + q^2 - pq)t \tag{19}$$

and (p, q, c) are arbitrary parameters. Figure 4 shows the characteristic dynamics described by the exact solution with $c = 0.1$, $p = 2$ and $q = 1$ for three subsequent time instances.

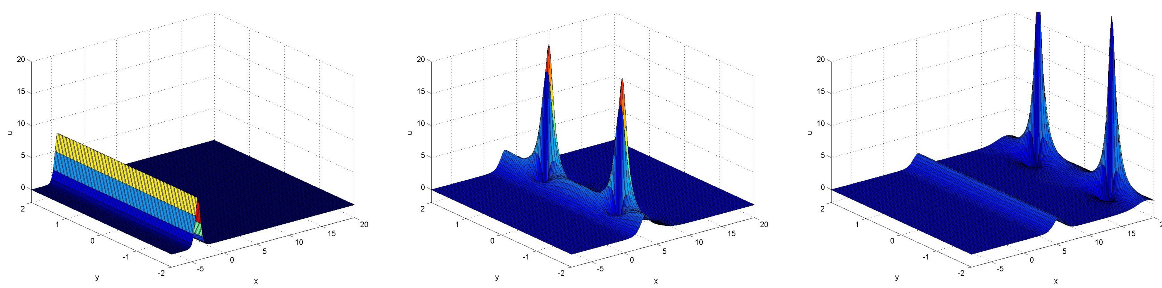

Figure 4: Transverse instability of a line soliton of the KPI equation.

Resonant interactions of line solitons in the KPII equation

One-dimensional solitary waves interacting under certain angles display non-trivial web patterns which resemble those observed in the ocean. The simplest such solution is the so-called "Miles resonance", or Y-shape soliton, which is obtained when the parameters of the three interacting soliton legs satisfy the resonance conditions $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$ and $\omega_1 + \omega_2 = \omega_3$ [Miles (1977); see also Newell and Redekopp (1977)]. A contour plot of such a solution is shown in Figure 5. Such a solution is the limiting case of an ordinary 2-soliton solution in which the asymptotic solitons satisfy the resonance condition.

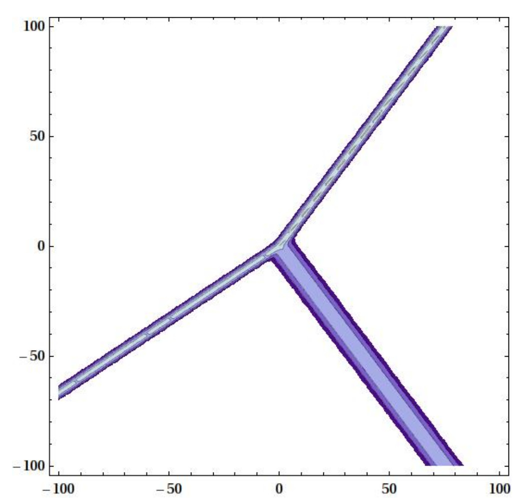

Figure 5: A resonant interaction between three line solitons of the KPII equation.

More general resonant solutions exhibiting web structure in the xy-plane were recently discovered by Biondini and Kodama (2003). A resonant 2-soliton solution, which is the simplest of such solutions, is shown in Figure 2 above.

For sufficiently small amplitudes and/or a sufficiently large angle between the two line solitons, their interaction is described by the regular Hirota 2-soliton solution. When the amplitudes and/or angle approaches the boundaries of the existence interval, however, the 2-soliton solution degenerates into the Miles solution shown in Figure 5, and, until recently, no solutions were known beyond this limit [e.g., see Infeld and Rowlands (2001)]. It is now known that, for amplitudes/angles beyond the Miles limit, the corresponding solution describes the resonant interaction of the two line solitons, such as the one shown in Figure 2 above (Biondini, 2007).

In general, all the multi-soliton solutions of KPII obtained from the tau-function (8) can be classified in terms of the number and characteristic parameters of the so-called "asymptotic solitons", namely, the soliton legs entering and exiting from a compact interaction domain and extending out to infinity either in the positive or in the negative y direction [see Kodama (2004), Biondini and Chakravarty (2006)]. All of these resonant multi-leg solutions of the KPII equation are believed to be stable with respect to perturbations [see Biondini (2007)], but no formal proof is available at present. A correspondence was also found to exist between these solutions and a special class of permutations (Chakravarty and Kodama, 2008).

Finite-genus and quasi-periodic solutions

Both variants of the KP equation admit a large family of exact periodic and quasi-periodic solutions. A finite-genus solution of KP with g phases is expressed in terms of a Riemann theta function $\theta(\mathbf{z}|B)$ as

$$u(x,y,t) = c + 2\partial_x^2 \ln\theta(\mathbf{z}_1 x + \mathbf{z}_2 y + \mathbf{z}_3 t + \mathbf{z}_0|B), \qquad (20)$$

where the constant c, the real g-dimensional vectors $\mathbf{z}_0,\ldots,\mathbf{z}_3$ and the $g\times g$ Riemann matrix B are determined by a compact connected Riemann surface of genus g and a set of g points (a divisor) on it [e.g., see Dubrovin (1981), Krichever (1989) and Belokolos *et al.* (1994)]. Explicitly, the theta function identified by a matrix B is defined in terms of its multi-dimensional Fourier series as

$$\theta(\mathbf{z}) = \sum_{\mathbf{m}\in\mathbb{Z}^g} \exp\left(\mathbf{m}^t B\mathbf{m} + i\mathbf{m}^t\cdot\mathbf{z}\right). \qquad (21)$$

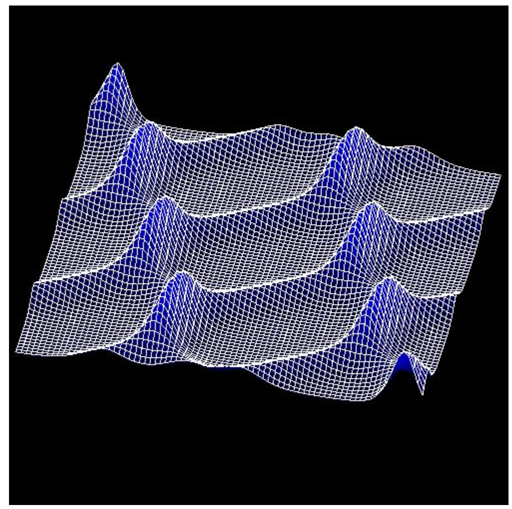

Figure 6: A genus-2 solution of the KPII equation (from the KP page).

Solutions obtained from (20) with $g = 1$ generalize to two spatial dimensions the cnoidal traveling-wave solutions of the Korteweg-de Vries equation. Solutions with $g = 2$ are also space-periodic, with a hexagonal period cell. For $g > 2$, however, they describe more complicated quasi-periodic wave patterns.

Finite-genus solutions of the KPII equation were found to accurately reproduce the wave patterns measured in laboratory experiments of 2-dimensional shallow water (Hammack *et al.*, 1989, 1995), and they describe certain wave patterns that are observed in the ocean. Figure 6 shows a genus-2 solution of the KPII equation. Various pictures of water wave patterns thought to be described well by the KPII equation and various pictures of actual KPII solutions are available online on the KP page maintained by Bernard Deconinck.

The formalism that describes finite-genus solutions of KP provided a solution of the century-old Schottky problem, which consisted in identifying all matrices B that are normalized period matrices of a genus-g Riemann surface. It was conjectured by Sergey Novikov that a matrix B is a period matrix if and only if Eq. (20) does generate a solution of KP. This conjecture was later proved by Shiota in 1986 (see also Debarre, 1995).

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