Solitary waves under intensity-dependent dispersion

Dmitry E. Pelinovsky

joint work with Panos Kevrekidis and Ryan Ross (University of Massachusetts, Amherst)

Introduction

The nonlinear Schrödinger equation has many applications in physics. It realizes the fundamental balance between nonlinearity and dispersion for propagation of nonlinear dispersive waves.

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Taking into account higher-order nonlinearity and dispersion gives an extended version of the NLS equation:

$$i\psi_t + \psi_{xx} + |\psi|^2\psi + ic_1\psi_{xxx} + ic_2|\psi|^2\psi_x + ic_3(|\psi|^2\psi)_x + c_4|\psi|^4\psi = 0.$$

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What we study is a different model where the dispersion coefficient depends on the wave intensity:

$$i\psi_t + (1 - |\psi|^2)\psi_{xx} = 0. \tag{NLS-IDD}$$

C.Y. Lin, J.H. Chang, G. Kurizki, and R.K. Lee, Optics Letters **45** (2020), 1471–1474

NLS with intensity-dependent dispersion

Two conserved quantities exist for NLS-IDD:

$$Q(\psi) = -\int_{\mathbb{R}} \log|1 - |\psi|^2| dx$$

and

$$E(\psi) = \int_{\mathbb{R}} |\psi_x|^2 dx.$$

Standing waves exist in the form $\psi(x,t) = e^{i\omega t} u(x)$ with (ω, u) satisfying

$$\omega u(x) = (1 - u^2) u''(x).$$

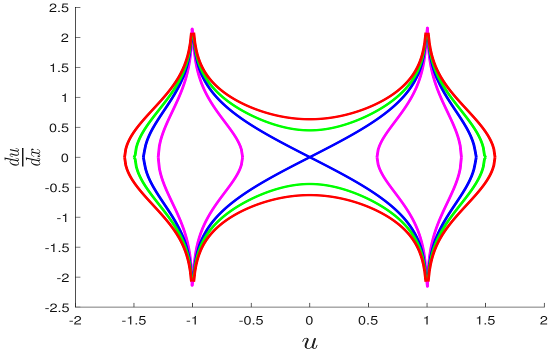
Solitary waves with $u(x) \to 0$ as $|x| \to \infty$ exist only if $\omega > 0$, in which case ω can be scaled out by $u(x) \mapsto u(\sqrt{\omega} x)$.

Phase plane portrait

Equation $(1-u^2)u'' = u$ is integrable with the first invariant:

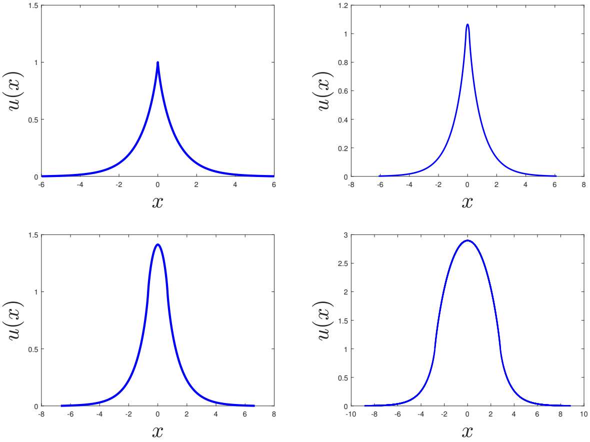
$$\frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\log|1-u^2| = C,$$

where C is constant. The solution is singular at $u = \pm 1$.



Possible solitary waves

Gluing the stable and unstable curves with another integral curves give a one-parameter family of single-humped solitary waves:



Top left: “cusped soliton". Bottom left: “bell-shaped soliton".

Questions on existence and stability of these solitary waves

- In what space (in what sense) do they exist?
- What is the nature of singularity at $u = \pm 1$?
- Can these solutions be characterized variationally?
- Are they stable in the time evolution of the NLS-IDD?

Existence result

Definition

We say that $u \in H^1(\mathbb{R})$ is a weak solution of the differential equation $u = (1 - u^2)u''$ if it satisfies the following equation

$$\langle u, \varphi \rangle + \langle (1 - u^2)u', \varphi' \rangle - 2\langle u(u')^2, \varphi \rangle = 0, \quad \text{for every } \varphi \in H^1(\mathbb{R}),$$

where $\langle \cdot, \cdot \rangle$ is the inner product in $L^2(\mathbb{R})$.

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where $\langle \cdot, \cdot \rangle$ is the inner product in $L^2(\mathbb{R})$.

Theorem (Ross–Kevrekidis–P, Q.Appl.Math. 79 (2021) 641)

There exists a one-parameter continuous family of weak, positive, and single-humped solutions of $u = (1 - u^2)u''$ parametrized by C.

What is needed for the proof beyond the phase plane analysis:

- ▷ $u \in H^1(\mathbb{R})$;
- ▷ $\lim\limits_{x \to x_0} (1 - u^2(x))u'(x) = 0$ for each x_0 where $u(x_0) = 1$.

Nature of singularity at $u = 1$

It follows from the first invariant

$$\frac{1}{2}\left(\frac{du}{dx}\right)^2 + \frac{1}{2}\log|1-u^2| = C,$$

that the cusped soliton is defined by the implicit function

$$|x| = \int_u^1 \frac{d\xi}{\sqrt{-\log(1-\xi^2)}}, \quad u \in (0,1).$$

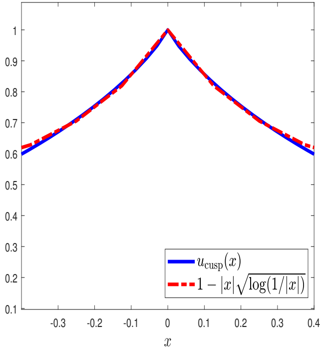
Asymptotic analysis [Alfimov–Korobeinikov–Lustri–P, Nonlinearity 32 (2019) 3445] gives as $|x| \to 0$:

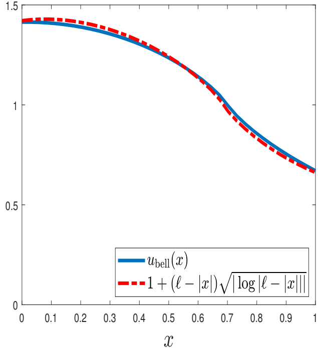
$$u(x) = 1 - |x|\sqrt{\log(1/|x|)}\left[1 + \mathcal{O}\left(\frac{\log\log(1/|x|)}{\log(1/|x|)}\right)\right].$$

Hence, $u'(x) \sim \sqrt{\log(1/|x|)}$ and $(1-u^2)u'(x) \sim |x|\log(1/|x|)$.

Numerical illustration of the asymptotic profile

$$u(x) = 1 - |x|\sqrt{\log(1/|x|)}\left[1 + \mathcal{O}\left(\frac{\log\log(1/|x|)}{\log(1/|x|)}\right)\right].$$





Left: “cusped soliton". Right: “bell-shaped soliton".

Towards the stability result

Recall the conserved quantities:

$$Q(\psi) = -\int_{\mathbb{R}} \log|1-|\psi|^2|dx, \quad E(\psi) = \int_{\mathbb{R}} |\psi_x|^2 dx.$$

Solitary wave $\psi(x,t) = u(x)e^{i\omega t}$ is a critical point of the action

$$\Lambda_\omega(u) = E(u) + \omega Q(u),$$

however, the formal expansion yields

$$\begin{aligned}\Lambda_\omega(u+\varphi) - \Lambda_\omega(u) = 2\langle u', \varphi'\rangle + 2\langle (1-u^2)^{-1}u, \varphi\rangle \\ + \mathcal{O}(\|\varphi'\|_{L^2}^2 + \|(1-u^2)^{-1}\varphi\|_{L^2\cap L^\infty}^2),\end{aligned}$$

which is not compatible with the definition of weak solutions:

$$u \in H^1(\mathbb{R}) : \quad \omega\langle u, \varphi\rangle + \langle (1-u^2)u', \varphi'\rangle - 2\langle u(u')^2, \varphi\rangle = 0,$$

for every $\varphi \in H^1(\mathbb{R})$.

New definition of weak solutions

Definition

Fix $L > 0$ and define

$$X_L := \left\{ u \in H^1(\mathbb{R}) : \ u(x) > 1, \ x \in (-L, L) \text{ and } u(x) \leq 1, \ |x| \geq L \right\}.$$

Pick $u_L \in X_L$ satisfying

$$\lim_{|x| \to L} \frac{u_L(x) - 1}{(L - |x|)\sqrt{|\log|L - |x|||}} = 1.$$

We say that $u \in X_L \subset H^1(\mathbb{R})$ is a weak solution if it satisfies the following equation

$$\langle u', \varphi' \rangle + \omega \langle (1 - u^2)^{-1} u, \varphi \rangle = 0, \quad \text{for every } \varphi \in H^1_L,$$

where $H^1_L := \left\{ \varphi \in H^1(\mathbb{R}) : \ (1 - u_L^2)^{-1} \varphi \in L^2(\mathbb{R}) \cap L^\infty(\mathbb{R}) \right\}$.

Stability result

Theorem (P–Ross–Kevrekidis, J. Phys. A 54 (2021) 445701)

For every $\mu > 0$ and $L > 0$, there exists a minimizer of the constrained variational problem

$$\mathcal{Q}_{\mu,L} := \inf_{u \in X_L} \{Q(u) : \quad E(u) = \mu\}.$$

The minimizer coincides with a scaled version of the one-parameter family of solitary waves for $C = C_{\mu L}$.

What is needed for the proof beyond the expansion of Λ_ω in X_L:

- Monotonicity of mappings $C \mapsto E(u_C)$ and $C \mapsto \ell_C$, where $2\ell_C$ is the length of the bell head;
- Scaling transformation;
- Convexity of action Λ_ω at u_C.

Monotonicity of mappings $C \mapsto E(u_C)$ and $C \mapsto \ell_C$

It follows from $(u')^2 + \log|1-u^2| = 2C$ that

$$E(u_C) = E(u_{\rm cusp}) + 2\int_1^{\sqrt{1+e^{2C}}} \sqrt{2C - \log(u^2-1)}du$$

$$\ell_C = \int_1^{\sqrt{1+e^{2C}}} \frac{du}{\sqrt{2C - \log(u^2-1)}}$$

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$\frac{dE(u_C)}{dC} > 0$ follows from

$$\frac{dE(u_C)}{dC} = 2\int_1^{\sqrt{1+e^{2C}}} \frac{du}{\sqrt{2C - \log(u^2-1)}} = 2\ell_C.$$

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$\frac{d\ell_C}{dC} > 0$ follows from a longer computation, which is similar to analysis of the period function for periodic orbits on the phase plane.

$$\begin{aligned} C\ell_C &= \int_1^{\sqrt{1+e^{2C}}} \frac{C du}{\sqrt{2C - \log(u^2-1)}} \\ &= \frac{1}{2}\int_1^{\sqrt{1+e^{2C}}} \left[\sqrt{2C - \log(u^2-1)} + \frac{\log(u^2-1)}{\sqrt{2C - \log(u^2-1)}} \right] du. \end{aligned}$$

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\end{aligned}
$$

Denote $A(u) := \log(u^2-1)$ and write $v^2 + A(u) = 2C$ for the integral curve at the constant level C. Since $A'(u) \neq 0$ for $u > 1$, we have

$$
\begin{aligned}
d\left[\frac{A(u)v}{A'(u)}\right] &= \left(1 - \frac{A(u)A''(u)}{[A'(u)]^2}\right) v du + \frac{A(u)}{A'(u)} dv \\
&= \left(1 - \frac{A(u)A''(u)}{[A'(u)]^2}\right) v du - \frac{A(u)}{2v} du.
\end{aligned}
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Integrating by parts yields

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\begin{aligned}
2C\ell_C &= \int_1^{\sqrt{1+e^{2C}}} \left[3 - \frac{2A(u)A''(u)}{[A'(u)]^2}\right] v du \\
&= \int_1^{\sqrt{1+e^{2C}}} \left[3 + \frac{1+u^2}{u^2} \log(u^2 - 1)\right] v du,
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After differentiating in C, we get

$$2C\frac{d\ell_C}{dC} = \int_1^{\sqrt{1+e^{2C}}} \left[1 + \frac{1+u^2}{u^2}\log(u^2-1)\right] \frac{du}{v},$$

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After another integration by parts,

$$\frac{d}{du}\left[\frac{u^2-1}{u}\sqrt{2C-\log(u^2-1)}\right] = -\frac{1}{\sqrt{2C-\log(u^2-1)}} + \frac{1+u^2}{u^2}v,$$

we finally obtain

$$\frac{d\ell_C}{dC} = \int_1^{\sqrt{1+e^{2C}}} \frac{(1+u^2)du}{u^2\sqrt{2C-\log(u^2-1)}} > 0.$$

Scaling transformation

Recall the variational problem for $\mu > 0$ and $L > 0$:

$$\mathcal{Q}_{\mu,L} := \inf_{u \in X_L} \{Q(u) : \quad E(u) = \mu\},$$

with the Euler–Lagrange equation $\omega u = (1 - u^2)u''$.

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with the Euler–Lagrange equation $\omega u = (1 - u^2)u''$.

Let u_C be a solution of $u = (1 - u^2)u''$. Then, $u_\omega(x) = u_C(\sqrt{\omega}x)$ is a solution of the Euler–Lagrange equation so that

$$Q(u_\omega) = \frac{1}{\sqrt{\omega}} Q(u_C), \quad E(u_\omega) = \sqrt{\omega} E(u_C)$$

and

$$L = \frac{1}{\sqrt{\omega}} \ell_C, \quad \mu = \sqrt{\omega} E(u_C).$$

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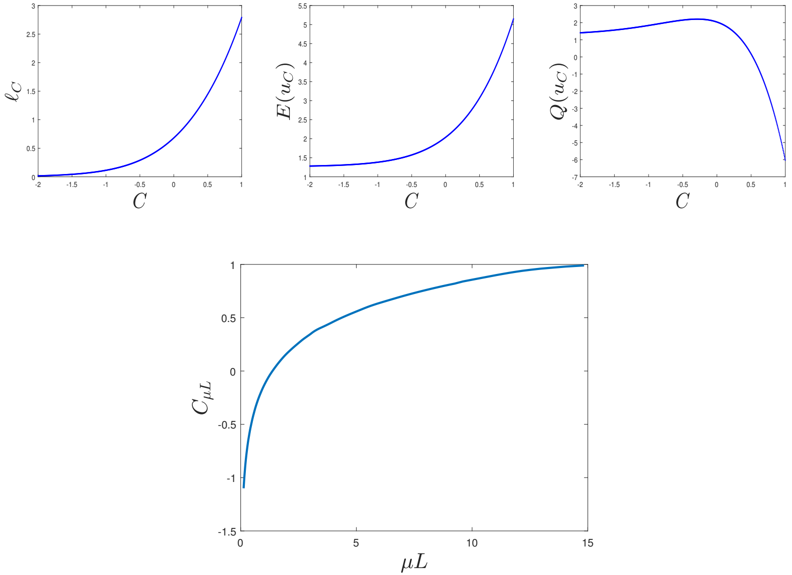
with the Euler–Lagrange equation $\omega u = (1 - u^2)u''$.

Transformation $(\omega, C) \mapsto (\mu, L)$ is invertible because the Jacobian is

$$\begin{vmatrix} \frac{\partial \mu}{\partial \omega} & \frac{\partial \mu}{\partial C} \\ \frac{\partial L}{\partial \omega} & \frac{\partial L}{\partial C} \end{vmatrix} = \frac{1}{2\omega} \left[E(u_C) \frac{d\ell_C}{dC} + \ell_C \frac{dE(u_C)}{dC} \right] > 0.$$

Hence the mapping $(\omega, C) \mapsto (\mu, L)$ is invertible and there exists a unique $C = C_{\mu,L}$ for every $\mu > 0$ and $L > 0$. In fact, $\ell_C E(u_C) = L\mu$.

Numerical illustrations



Convexity of action Λ_ω

Let $v + iw$ with real $v, w \in H^1_{\ell_C} \subset H^1(\mathbb{R})$ be a perturbation to u_C. Then, the action is expanded as

$$\Lambda_{\omega=1}(u_C + v + iw) = \Lambda_{\omega=1}(u_C) + Q_+(v) + Q_-(w) + R(v,w),$$

where $R(v,w)$ is the remainder term

$$R(v,w) = \int_{\mathbb{R}} \left[\log\left(1 - \frac{2u_C v + v^2 + w^2}{1-u_C^2}\right) + \frac{2u_C v}{1-u_C^2} + \frac{(1+u_C^2)v^2}{(1-u_C^2)^2} + \frac{w^2}{1-u_C^2} \right] dx.$$

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$R(v,w)$ is cubic with respect to perturbation:

$$|R(v,w)| \leq C\|(1-u_C^2)^{-1}v\|^3_{L^2\cap L^\infty} + C\|(1-u_C^2)^{-1}w\|^3_{L^2\cap L^\infty},$$

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$$\Lambda_{\omega=1}(u_C + v + iw) = \Lambda_{\omega=1}(u_C) + Q_+(v) + Q_-(w) + R(v, w),$$

whereas Q_+ and Q_- are the quadratic forms:

$$Q_+(v) = \int_{\mathbb{R}} \left[(v_x)^2 + \frac{(1+u_C^2)v^2}{(1-u_C^2)^2} \right] dx, Q_-(w) = \int_{\mathbb{R}} \left[(w_x)^2 + \frac{w^2}{1-u_C^2} \right] dx,$$

For cusped soliton with $0 < u \leq 1$, they are coercive and bounded as

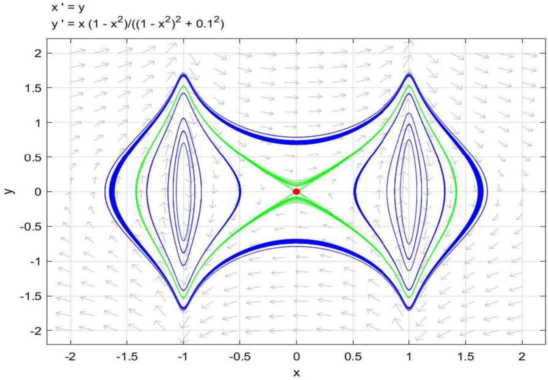
$$Q_\pm(v) \geq \|v\|^2_{H^1}, \quad Q_\pm(v) \leq C_\pm \left(\|v'\|^2_{L^2} + \|(1-u_C^2)^{-1} v\|^2_{L^2} \right)$$

Hence $u_{C_{\mu L}}$ is a minimizer of $Q(u)$ in X_L for fixed $L > 0$ and $\mu > 0$.

Numerical methods: regularization

Fix $\epsilon > 0$ and replace $u = (1 - u^2)u''$ by

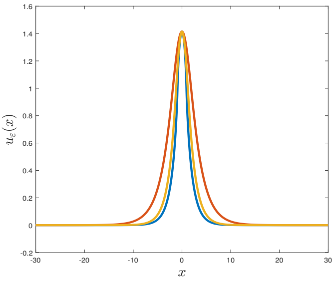
$$u_\epsilon'' = \frac{u_\epsilon(1 - u_\epsilon^2)}{(1 - u_\epsilon^2)^2 + \epsilon^2},$$

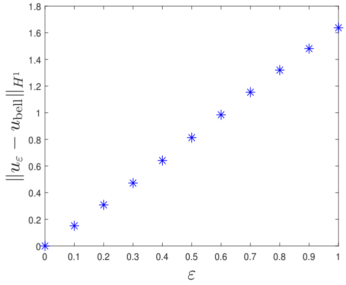


Numerical methods: regularization

Only bell soliton $u_{C=0}$ can be recovered by using this numerical method. Moreover, we have proved that

$$\|u_\epsilon - u_{C=0}\|_{H^1} \to 0 \quad \text{as} \quad \epsilon \to 0.$$





Numerical methods: Petviashvili iterations

Rewrite $u = (1 - u^2)u''$ as $u - u'' = -u^2 u''$ and interpret a solution $u \in H^1(\mathbb{R})$ as a fixed point $u = T(u)$ of the nonlinear operator

$$T(u) := -(1 - \partial_x^2)^{-1} u^2 \partial_x^2 u.$$

The fixed point can be approached by iterations $\{w_n\}_{n \in \mathbb{N}}$ of the Petviashvili's method

$$w_{n+1} = -\lambda(w_n)^{3/2} (1 - \partial_x^2)^{-1} w_n^2 \partial_x^2 w_n,$$

where

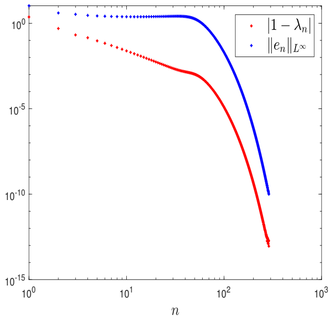
$$\lambda(w) := \frac{\int_{\mathbb{R}} (w^2 + w_x^2) dx}{3 \int_{\mathbb{R}} w^2 w_x^2 dx}.$$

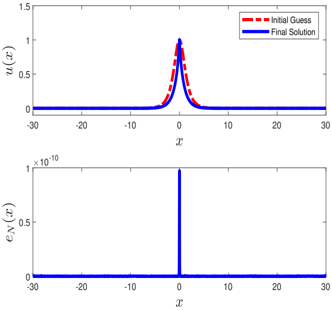
If $u \in H^1(\mathbb{R})$ is a fixed point of $T(u)$, then $\lambda(u) = 1$.

Numerical methods: Petviashvili iterations

Only cusped soliton u_{cusp} can be recovered by using this numerical method. Moreover, we have proved that if w_0 is in a local neighborhood of u_{cusp}, then

$$\|w_n - u_{\text{cusp}}\|_{H^1} \to 0 \quad \text{as} \quad n \to \infty.$$





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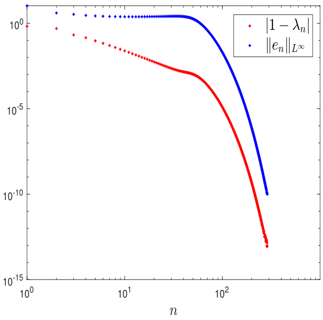
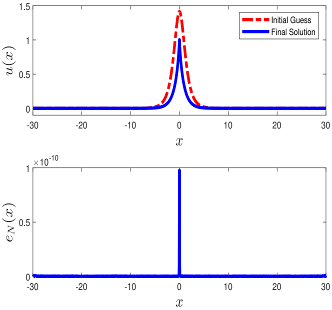




Numerical methods: Newton iterations

Represent a solution of $u = (1-u^2)u''$ as a root of the nonlinear equation $F(u) = 0$, where $F(u) := -(1-u^2)\partial_x^2 u + u$. Roots of the nonlinear equation $F(u) = 0$ in $H^1(\mathbb{R})$ can be approximated by using the Newton iterations:

$$u_{n+1} = u_n - \mathcal{L}^{-1} F(u_n),$$

where $\mathcal{L} := -(1-u^2)\partial_x^2 + \frac{1+u^2}{1-u^2}$.

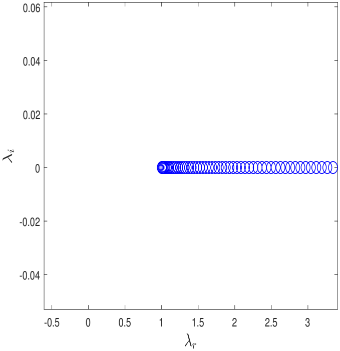
Let $u = u_{\rm cusp}$ be the cusped soliton and $v \in H^1(\mathbb{R})$ satisfy $v(0) = 0$. Then,

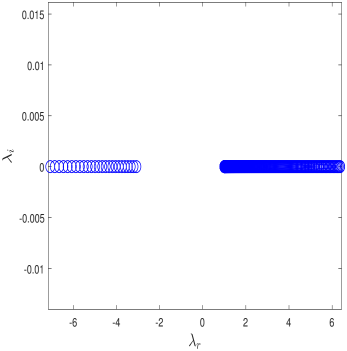
$$\begin{aligned}\langle \mathcal{L}v, v\rangle &= \int_{\mathbb{R}} (1-u^2)(v')^2 dx + \int_{\mathbb{R}} (uu')' v^2 dx + \int_{\mathbb{R}} \frac{1+u^2}{1-u^2} v^2 dx \\ &= \int_{\mathbb{R}} (1-u^2)(v')^2 dx + \int_{\mathbb{R}} \left[\frac{1+2u^2}{1-u^2} + (u')^2\right] v^2 dx,\end{aligned}$$

hence $\sigma(\mathcal{L}) \geq 1$.

Numerical methods: Newton iterations

All solitary waves of the family u_C can be recovered by using this numerical method.





Numerical methods: Newton iterations

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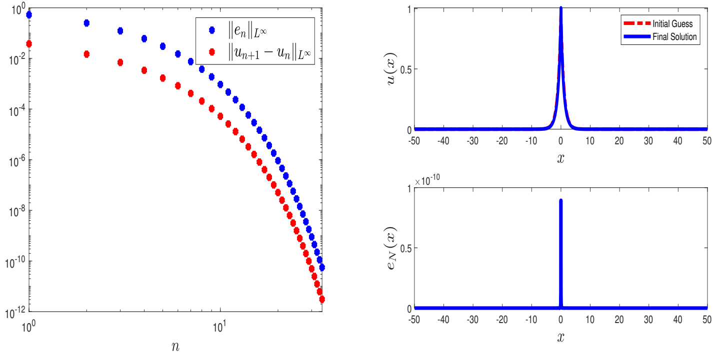

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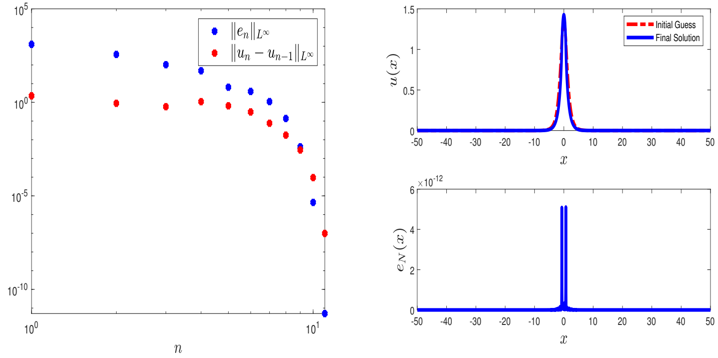

Summary

We considered NLS equation with intensity-dependent dispersion

$$i\psi_t + (1 - |\psi|^2)\psi_{xx} = 0.$$

- Continuum of singular solitary waves exists $\psi(x,t) = u_C(x)e^{it}$.
- Each solitary wave can be characterized as a minimizer of mass for fixed energy μ and distance L between two singularities.
- These solitary waves are robust in the numerical simulations.
- Well-posedness and stability theory are opened for studies.

Related problems: compactons

KdV equation with sublinear nonlinearity:

$$u_t - \alpha|u|^{\alpha-1}u_x + u_{xxx} = 0, \quad \alpha \in (0,1).$$

It admits compactly supported solutions (compactons) in the form

$$u(t,x) = a\sin^{\frac{2}{1-\alpha}}(x-ct), \quad 0 \leq x - ct \leq \pi,$$

with some uniquely specified a and c.

D.E. Pelinovsky, A.V. Slunyaev, A.V. Kokorina, and E.N. Pelinovsky, Comm. Nonlinear Sci. Numer. Simul. **101** 105855 (2021)

Similar study of stability of compactons in related problems:

P. Germain, B. Harrop–Griffiths, J. Marzuola, Q. Appl. Math. **78** (2020), 1538

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Thank you for your attention!