MATH 3J04: Solutions to Home Assignment # 3

Problem 19.1 #6: The improved Euler method for the logistic equation is

$$y^*_{n+1} = y_n + \Delta t \left[y_n - y_n^2 \right],$$

$$y_{n+1} = y_n + 0.5\Delta t \left[y_n - y_n^2 + y^*_{n+1} - y^{*2}_{n+1} \right],$$

starting with $y_0 = 0.5$ and $\Delta t = 0.1$. The numerical approximations are $y_1 = 0.525$, $y_2 = 0.550$, $y_3 = 0.574$, $y_4 = 0.599$, $y_5 = 0.622$, $y_6 = 0.646$, $y_7 = 0.668$, $y_8 = 0.690$, $y_9 = 0.711$.

Problem 19.3 #4: The Euler method for the given second-order equation is

$$(y_1)_{n+1} = (y_1)_n + \Delta t (y_2)_n,$$

$$(y_2)_{n+1} = (y_2)_n + \Delta t \left[t_n (y_2)_n - 3(y_1)_n \right],$$

starting with $(y_1)_0 = 0$, $(y_2)_0 = -3$, and $\Delta t = 0.05$. Here $y_1 = y(t)$ and $y_2 = \dot{y}$. The numerical approximations for $y(t_n)$ are $y_1 = -0.150$, $y_2 = -0.300$, $y_3 = -0.449$, $y_4 = -0.597$, $y_5 = -0.742$. The local error (defined as a deviation from the exact solution) is $e_1 = 0.0001$, $e_2 = 0.0001$, $e_3 = 0.0024$, $e_4 = 0.0050$, $e_5 = 0.0080$.

Problem 19.3 #8: Rewrite the Bessel equation as a system

$$\frac{dy_1}{dt} = y_2, \quad \frac{dy_2}{dt} = -y_1 - \frac{y_2}{t}$$

and write the classic Runge method. The numerical approximations are

t_n	1.5	2	2.5	3	3.5	4	4.5	5
$(y_1)_n$	0.5119	0.2240	−0.0483	−0.2601	−0.3803	−0.3975	−0.3211	−0.1782
$(y_2)_n$	−0.5580	−0.5769	−0.4974	−0.3394	−0.1378	0.0657	0.2309	0.3277

Problem 10.3 #6: The Fourier series is

$$f(x) = 1 - x^2 = \frac{2}{3} - \frac{4}{\pi^2} \sum_{n+1}^{\infty} \frac{\cos(\pi n)}{n^2} \cos(\pi n x)$$

Problem 10.8 #16: The Fourier sine tranform is

$$f(x) = \begin{cases} \pi - x, & 0 < x < \pi \\ 0, & x > \pi \end{cases} = \int_0^\infty B(\omega) \sin(\omega x) d\omega,$$

where

$$B(\omega) = \frac{2(\pi\omega - \sin(\pi\omega))}{\pi\omega^2}$$

Problem 10.10 #8: The Fourier transform is

$$f(x) = e^{-|x|} = \int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega,$$

where

$$F(\omega) = \frac{1}{\pi(1+\omega^2)}.$$