# MATH 4X03: Home Assignment # 2

**Due to:** October 10, 2000

**Problem 1**: Find the radius of convergence of the following series:

$$\text{(a)} \quad \sum_{n=0}^{\infty} e^{-nz}, \qquad \text{(b)} \quad \sum_{n=0}^{\infty} \frac{z^n}{(n+1)!}, \qquad \text{(c)} \quad \sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$$

Which of those series are Taylor series?

**Problem 2**: Find Taylor series expansion around $z = 0$ of the function:

$$f(z) = \frac{e^{z^2} - 1 - z^2}{z^3}$$

What is the radius of convergence of the Taylor series?

**Problem 3**: Find Taylor series expansion around $z = 1$ of the function: $f(z) = 1/z$. What is the radius of convergence of the Taylor series? Use the Taylor series of the function $f(z) = 1/z$ to find Taylor series of the function $g(z) = 1/z^2$ near $z = 1$.

**Problem 4**: Given the function

$$f(z) = \frac{z}{(z-2)(z+i)}$$

expand $f(z)$ in a Laurent series in powers of $z$ in the regions:

$$\text{(a)} \quad |z| < 1, \qquad \text{(b)} \quad 1 < |z| < 2, \qquad \text{(c)} \quad 2 < |z|.$$

**Problem 5**: Show that the functions are meromorphic, that is, the only singularities in the finite $z$ plane are poles. Determine the location, order and strength of the poles.

$$\text{(a)} \quad \frac{z}{z^4 + 4}, \qquad \text{(b)} \quad \tan z, \qquad \text{(c)} \quad \frac{z}{sin^2 z}.$$