MATH 4X03: Home Assignment # 3

Due to: October 24, 2000

Problem 1: Evaluate the following integrals:

$$\text{(a)} \quad \int_0^\infty \frac{dx}{x^6+1}, \qquad \text{(b)} \quad \int_{-\infty}^\infty \frac{\cos kx \cos mx}{x^2+a^2} dx$$

where k, m, a are real numbers.

Problem 2: Use principal value integrals to evaluate the following integrals:

$$\text{(a)} \quad \int_0^\infty \frac{\cos kx - \cos mx}{x^2} dx, \qquad \text{(b)} \quad \int_0^\infty \frac{\sin x}{x(x^2+1)} dx$$

Problem 3: Suppose the functions $f(z)$ and $g(z)$ are holomorphic everywhere outside the circle C_R of radius R centered at the origin, with the limits:

$$\lim_{z\to\infty} f(z) = f_\infty, \qquad \lim_{z\to\infty} zg(z) = g_\infty,$$

where f_∞ and g_∞ are complex constants. Find

$$\frac{1}{2\pi i}\int_{C_R} g(z) e^{f(z)} dz$$

Problem 4: The Poisson formula for the harmonic function $u(r,\theta)$ at the unit disc is

$$u(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} U(\phi) \frac{1-r^2}{1-2r\cos(\theta-\phi)+r^2} d\phi,$$

where $U(\theta) = \lim_{r\to 1^-} u(r,\theta)$. Derive the following expression for the harmonic conjugate function $v(r,\theta)$ at the unit disc:

$$v(r,\theta) = v(0) + \frac{1}{\pi}\int_0^{2\pi} U(\phi) \frac{r\sin(\theta-\phi)}{1-2r\cos(\theta-\phi)+r^2} d\phi.$$

What is the limiting function $V(\theta)$:

$$V(\theta) = \lim_{r\to 1^-} v(r,\theta) \quad ?$$