MATH 4X03: Home Assignment # 4

Due to: November 7, 2000

Problem 1: Find a linear transformation that maps the circle $C_z : |z-1| = 1$ onto the circle $C_w : |w - 3i/2| = 2$.

Problem 2: Show that the function $w = e^z$ maps the interior of the rectangle:

$$R_z = \{z : 0 < \mathrm{Re}(z) < 1,\ 0 < \mathrm{Im}(z) < 2\pi\}$$

onto the interior of the annulus:

$$R_w = \{w : 1 < |w| < e\},$$

which has a jump along the positive real axis.

Problem 3: Find the image of the triangular region:

$$D_z = \{z : \mathrm{Re}(z) > 0,\ \mathrm{Im}(z) > 0,\ \mathrm{Re}(z) + \mathrm{Im}(z) < 1\}$$

under the mapping $w = z^3$.

Problem 4: Show that the transformation $w = 2z^{-1/2} - 1$ maps the (infinite) domain exterior of the parabola $y^2 = 4(1-x)$ conformally onto the domain $|w| < 1$. Explain why this transformation does not map the (infinite) domain interior of the parabola conformally onto the domain $|w| > 1$.

Problem 5: Show that the transformation $w = (z-a)/(z+a)$, where $a = \sqrt{c^2 - \rho^2}$, c and ρ are real, and $0 < \rho < c$, maps the multiple domain

$$D_z = \{z : |z - c| < \rho \cup \mathrm{Re}(z) < 0\}$$

onto the annular multiple domain

$$D_w = \{w : |w| < \delta \cup |w| > 1\}$$

Find the radius δ of the inner circle.

Problem 6: Show that the transformation $w_1 = [(1+z)/(1-z)]^2$ maps the upper half unit circle onto the upper half plane. Show then that the transformation $w_2 = (w_1 - i)/(w_1 + i)$ maps the upper half plane onto the interior of the unit circle. Find an elementary conformal mapping that maps a semicircular disk onto a full disk.